Scales of nonlinear relaxation and the problem of balance of wind-driven seas

Sergei Badulin[1,2], Vladimir Zakharov[2,3,4]

[1]P.P. Shirshov Institute of Oceanology, Russian Academy of Science
[2]Novosibirsk State University, Russia
[3]Arizona University, USA
[4]P. N. Lebedev Physical Institute of the Russian Academy of Science

European Geosciences Union General Assembly 2011
Vienna, Austria, 7 April 2011

Klauss HASSELMANN 1962, On the nonlinear energy transfer in a gravity wave spectrum

Kinetic equation

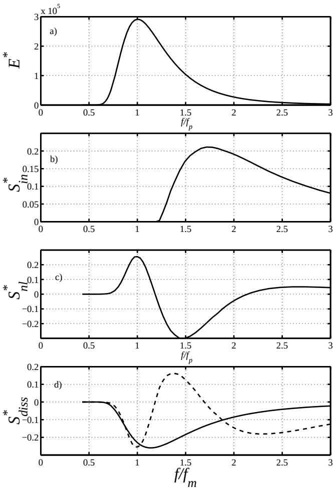
$$\frac{\partial n_{\mathbf{k}}}{\partial t} + \nabla_{\mathbf{k}} \omega_{\mathbf{k}} \nabla_{\mathbf{r}} n_{\mathbf{k}} = S_{in}\left[n_{\mathbf{k}}\right] + S_{diss}\left[n_{\mathbf{k}}\right] + S_{nl}\left[n_{\mathbf{k}}\right]$$

$n(\mathbf{k})$ – spatial spectrum of wave action (Fourier amplitudes squared for deep water $kh \gg 1$)

Important! Right-hand side

- Wave input S_{in} – empirico-heuristical
- Dissipation S_{diss} – empirico-heuristical
- Nonlinear transfer S_{nl} – from "the first principles"

Komen, Hasselmann, Hasselmann 1984, “On the existence of a fully developed wind-sea spectrum”



- Pierson-Moskowitz spectrum
- Input by Snyder et al. (1981)

  $S_{in}(\omega) =$
  $\max\left(0, 0.25\rho_a/\rho_w\omega\right.$
  $\left.\times(28u_*/C_p - 1)\right);$
- Dissipation

  $S_{diss} = -S_{in} - S_{nl}$

Output

$$\tilde{S}_{in} : \tilde{S}_{nl} : \tilde{S}_{diss} = 3 : (-1) : (-2)$$

$$\tilde{S}_i = \int_0^{2.5f_m} S_i df d\theta$$

Q. What is responsible for wind-wave balance?

Answers

1. Mainstream
   Wave input and dissipation provide a relaxation to an inherent state

2. Non-conventional ?
   Conservative nonlinear transfer term
   contains both forcing and damping
   and is able to provide the strong relaxation on its own ! ! !

Nonlinear forcing and damping

$$S_{nl} = \pi g^2 \int |T_{0123}|^2 \tag{1}$$
$$(N_1 N_2 N_3 + N N_{\mathbf{k}_2} N_{\mathbf{k}_3} - N N_{\mathbf{k}_1} N_{\mathbf{k}_2} - N N_{\mathbf{k}_1} N_{\mathbf{k}_3})$$
$$\times \delta(\mathbf{k} + \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3)\delta(\omega_{\mathbf{k}} + \omega_1 - \omega_2 - \omega_3) d\mathbf{k}_1 d\mathbf{k}_2 d\mathbf{k}_3$$

Split into two terms

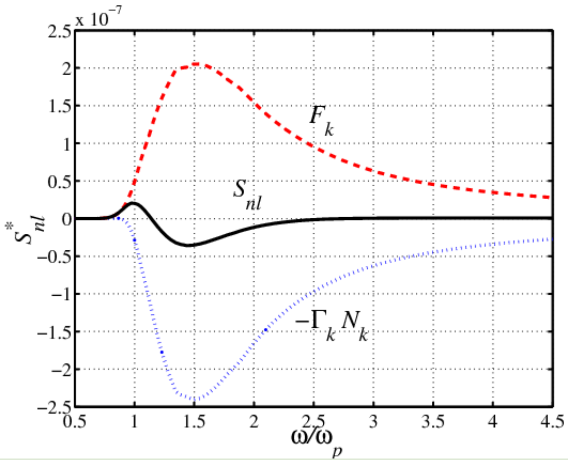
$$S_{nl} = F_{\mathbf{k}} - \Gamma_{\mathbf{k}} N_{\mathbf{k}} \tag{2}$$

where

$$F_{\mathbf{k}} = \pi g^2 \int |T_{0123}|^2 N_1 N_2 N_3 \tag{3}$$
$$\times \delta(\mathbf{k} + \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3)\delta(\omega_{\mathbf{k}} + \omega_1 - \omega_2 - \omega_3) d\mathbf{k}_1 d\mathbf{k}_2 d\mathbf{k}_3$$

$$\Gamma_{\mathbf{k}} = \pi g^2 \int |T_{0123}|^2 (N_1 N_2 + N_1 N_3 - N_2 N_3) \tag{4}$$
$$\times \delta(\mathbf{k} + \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3)\delta(\omega_{\mathbf{k}} + \omega_1 - \omega_2 - \omega_3) d\mathbf{k}_1 d\mathbf{k}_2 d\mathbf{k}_3$$

Split S_{nl} into two terms (N.N. Ivenskikh approach based on WRT-algorithm)



S_{nl} is small due to proximity to an inherent state !

Theoretical estimate of Γ_{nl}

Hypothesis: The key contribution to Γ_{nl} is from interactions of pairs of long and short waves

$$\Gamma_{\mathbf{k}} \approx 2\pi g^2 \int |T_{0103}|^2 \delta(\omega_0 - \omega_3) N_1 N_3 d\mathbf{k}_1 d\mathbf{k}_3 \qquad (5)$$

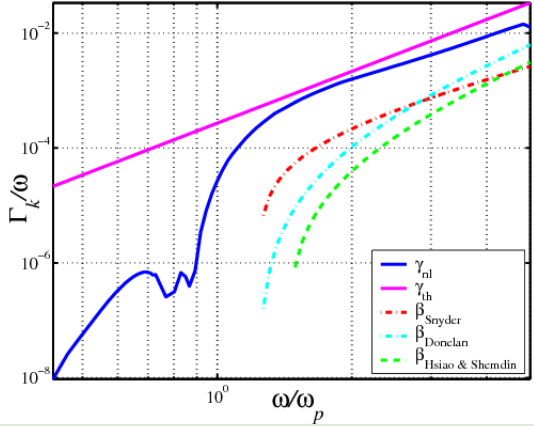
$$T_{0103} \approx 2|\mathbf{k}_1|^2 |\mathbf{k}| \cos\Theta$$

$$\Gamma_{\mathbf{k}} = 36\pi\omega(\omega/\omega_p)^3 \mu_p^4 \cos^2\Theta, \qquad (6)$$

small parameter $\mu_p = \sqrt{\frac{E\omega_p^4}{g^2}}$ – wave steepness

An enhancing factor: $36\pi \approx 113.1$

Compare nonlinear damping decrement and wind input increment



S_{nl} surpasses S_{in} and S_{diss} in order of magnitude !

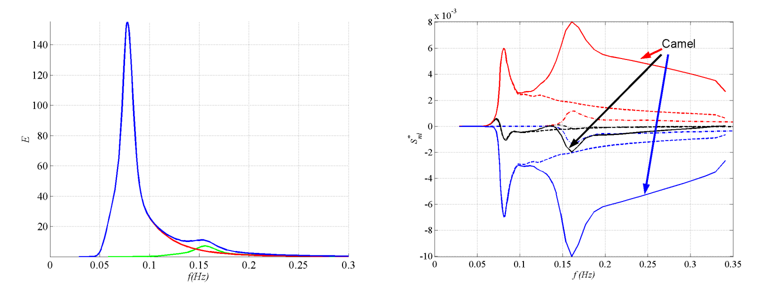
An example: Mixed sea

How to use our simple estimate of nonlinear damping?

$$\Gamma_{\mathbf{k}} = 36\pi\omega(\omega/\omega_p)^3 \mu_p^4 \cos^2\Theta,$$

High (ω/ω_p) – from swell peak, high μ_p – from wind waves

I. Young, 2006, JGR



See Badulin et al. 2008, Rogue Waves 2008

Summary

1. Nonlinear relaxation, generally, is much stronger than quasi-linear external forcing and wave dissipation;
2. Interactions of long and short waves play key role in this relaxation;
3. We do not ignore wave input and dissipation, we just put them into proper place