ONCE MORE ON THE DOMINANCE OF NONLINEAR TRANSFER

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WISE 2011, Quindao, China, 22 May 2011

Outline

Klauss HASSELMANN 1962, “On the nonlinear energy transfer in a gravity wave spectrum”

Kinetic equation

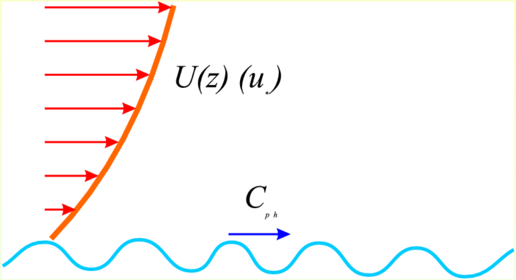
$$\frac{\partial n_{\mathbf{k}}}{\partial t} + \nabla_{\mathbf{k}}\omega_{\mathbf{k}}\nabla_{\mathbf{r}} n_{\mathbf{k}} = S_{in}\left[n_{\mathbf{k}}\right] + S_{diss}\left[n_{\mathbf{k}}\right] + S_{nl}\left[n_{\mathbf{k}}\right]$$

$n(\mathbf{k})$ – spatial spectrum of wave action (Fourier amplitudes squared for deep water $kh \gg 1$)

Important! Right-hand side

- Wave input S_{in} – empirico-heuristical
- Dissipation S_{diss} – empirico-heuristical
- Nonlinear transfer S_{nl} – from “the first principles”

Wave input. The Vavilov-Cherenkov excitation Cherenkov, Frank, Tamm – Nobel Prize 1958



Quasi-linea wave input

$S_{in} = \beta(\mathbf{k}, N_{\mathbf{k}})N_{\mathbf{k}},$

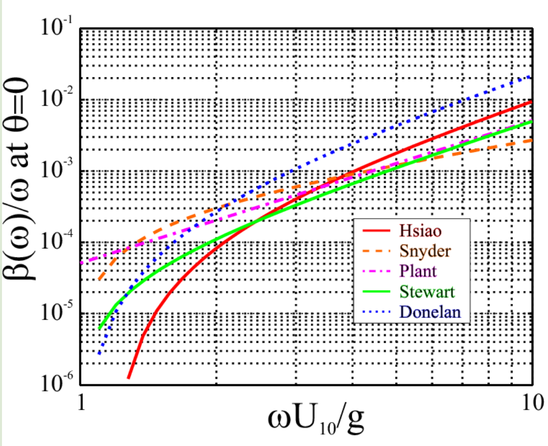
wave growth $\beta(\mathbf{k}) = \varrho\omega(\mathbf{k})(\varsigma - 1)^n \quad \text{at } \varsigma > 1,\ n = 1, 2.$

The Cherenkov-like factor

$\varsigma = s\dfrac{U_h}{C_{ph}}\cos\theta$, $s = O(1)$, θ is wave-to-wind direction.

What is "true wave input"?

What growth rate is correct?



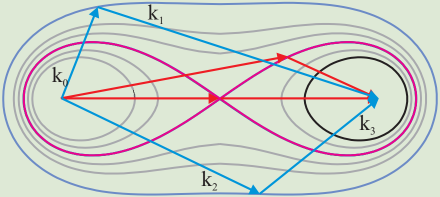
Growth rate parameterizations used in wave modeling

Hasselmann equation. Collision integral

$$S_{nl}[N_{\mathbf{k}}] = \int_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3} |T_{\mathbf{k},\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3}|^2 \{N_2 N_3(N+N_1) - N N_1(N_2+N_3)$$
$$\times\delta(\mathbf{k}+\mathbf{k}_1-\mathbf{k}_2-\mathbf{k}_3)\delta(\omega+\omega_1-\omega_2-\omega_3)d\mathbf{k}_1 d\mathbf{k}_2 d\mathbf{k}_3$$

4-wave resonances

$$\begin{cases} \omega_0 + \omega_1 = \omega_4 + \omega_3 \\ \mathbf{k}_0 + \mathbf{k}_1 = \mathbf{k}_4 + \mathbf{k}_3 \end{cases}$$



Explicit formula for T_{0123}

$$T_{1234} = -\frac{1}{32\pi^2}\frac{1}{(q_1q_2q_3q_4)^{1/4}}$$
$$\times\Big\{(-\boldsymbol{k}_2\boldsymbol{k}_3 + q_2q_3)(-\boldsymbol{k}_1\boldsymbol{k}_4 + q_1q_4)$$
$$+ (-\boldsymbol{k}_1\boldsymbol{k}_3 + q_1q_3)(-\boldsymbol{k}_2\boldsymbol{k}_4 + q_2q_4)$$
$$+ (\boldsymbol{k}_1\boldsymbol{k}_2 + q_1q_2)(\boldsymbol{k}_3\boldsymbol{k}_4 + q_3q_4)$$
$$+ \omega_1\omega_2\omega_3\omega_4$$
$$\times\Big[q_1^2 + q_2^2 + q_3^2 + q_4^2 - q_{1-3}(\omega_2-\omega_4)^2$$
$$-q_{2-3}(\omega_2-\omega_3)^2 - q_{1+2}(\omega_3+\omega_4)^2\Big]$$
$$+ \frac{(\omega_2-\omega_4)^2}{q_{1-3}-(\omega_2-\omega_4)^2}$$
$$\times[2\boldsymbol{k}_1\boldsymbol{k}_3 + \omega_1\omega_3(q_1+q_3-q_{1-3})]$$
$$\times[2\boldsymbol{k}_2\boldsymbol{k}_4 + \omega_2\omega_4(q_2+q_4-q_{1-3})]$$
$$+ \frac{(\omega_2-\omega_3)^2}{q_{2-3}-(\omega_2-\omega_3)^2}$$
$$\times[2\boldsymbol{k}_1\boldsymbol{k}_4 + \omega_1\omega_4(q_1+q_4-q_{2-3})]$$
$$\times[2\boldsymbol{k}_2\boldsymbol{k}_3 + \omega_2\omega_3(q_2+q_3-q_{2-3})]$$
$$+ \frac{(\omega_3+\omega_4)^2}{q_{1+2}-(\omega_3+\omega_4)^2}$$
$$\times[2\boldsymbol{k}_1\boldsymbol{k}_2 + \omega_1\omega_2(q_1+q_2-q_{1+2})]$$

Nonlinear transfer in spectral wave forecasting models

L. Cavaleri et al. / Progr. in Oceanogr. 75 (2007) 603-674

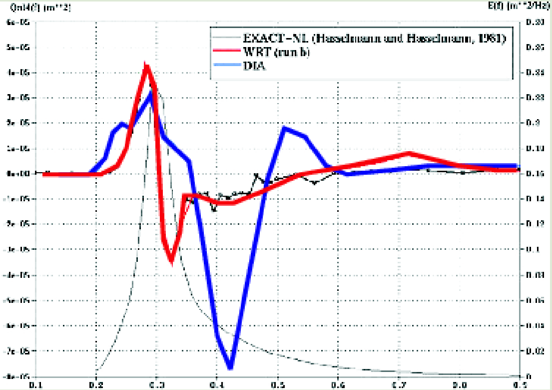
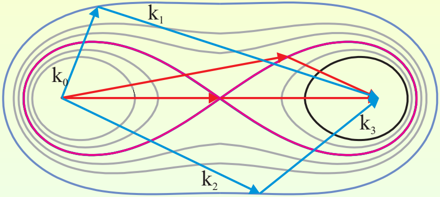


Fig. 7. Nonlinear transfer term Qnl4(f) computed for the test wave spectrum in deep water. These curves are obtained by integration over wave directions.

Direct Interaction Approximation for the collision integral S_{nl}

DIA takes into account just 4 (four!) symmetric quadruplets from all the continuum (!?)

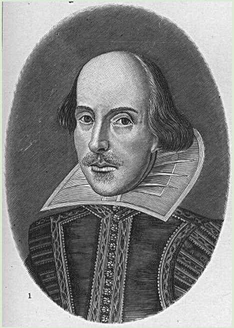


The modeling within the approach requires solution of great number of equations (say, 24(angles)×25(frequencies)) for spatial spectra while only two characteristics of the wave field (H_s, T_s – significant height and period) are really used in wave forecasting.

Summary of the section

"Something is rotten in the state of Denmark"

William Shakespeare

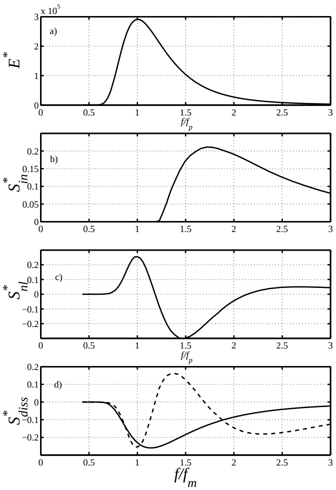


A misguiding star of wind-wave modeling

by Komen, Hasselmann, Hasselmann 1984

Komen, Hasselmann, Hasselmann 1984,

“On the existence of a fully developed wind-sea spectrum”



- Pierson-Moskowitz spectrum
- Input by Snyder et al. (1981)

  $S_{in}(\omega) =$
  $\max\left(0, 0.25\rho_a/\rho_w\omega\right.$
  $\left.\times(28u_*/C_p - 1)\right);$
- Dissipation
  $S_{diss} = -S_{in} - S_{nl}$

Output

$$\tilde{S}_{in} : \tilde{S}_{nl} : \tilde{S}_{diss} = 3 : (-1) : (-2)$$

$$\tilde{S}_i = \int_0^{2.5 f_m} S_i \, df d\theta$$

Q. What is responsible for wind-wave balance?

Answers

1. Mainstream
   Wave input and dissipation provide a relaxation to an inherent state

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Answers

1. Mainstream
   Wave input and dissipation provide a relaxation to an inherent state

2. Non-conventional ?
   Conservative nonlinear transfer term
   contains both forcing and damping
   and is able to provide the strong relaxation on its own ! ! !

Nonlinear forcing and damping

$$S_{nl} = \pi g^2 \int |T_{0123}|^2 \tag{1}$$
$$(N_1 N_2 N_3 + N N_{\mathbf{k}_2} N_{\mathbf{k}_3} - N N_{\mathbf{k}_1} N_{\mathbf{k}_2} - N N_{\mathbf{k}_1} N_{\mathbf{k}_3})$$
$$\times \delta(\mathbf{k} + \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3)\delta(\omega_{\mathbf{k}} + \omega_1 - \omega_2 - \omega_3)\mathrm{d}\mathbf{k}_1 \mathrm{d}\mathbf{k}_2 \mathrm{d}\mathbf{k}_3$$

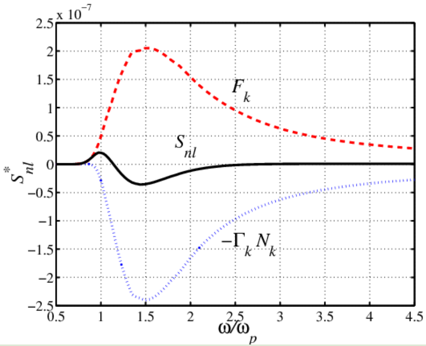
Split into two terms

$$S_{nl} = F_{\mathbf{k}} - \Gamma_{\mathbf{k}} N_{\mathbf{k}} \tag{2}$$

where

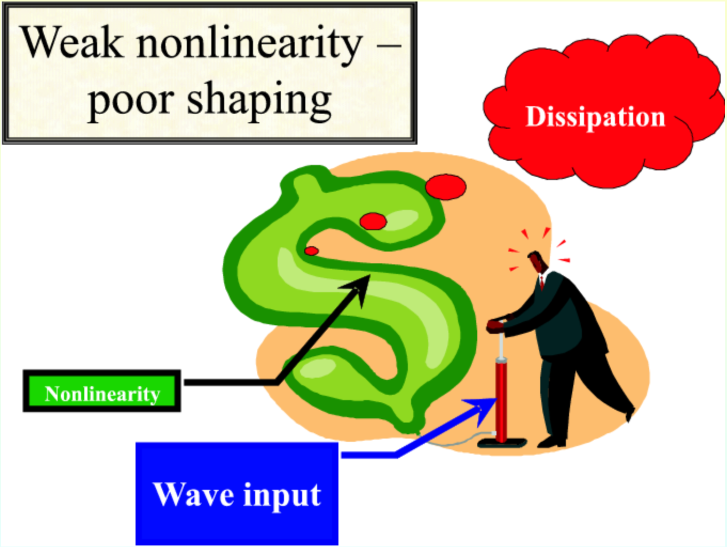
$$F_{\mathbf{k}} = \pi g^2 \int |T_{0123}|^2 N_1 N_2 N_3 \tag{3}$$
$$\times \delta(\mathbf{k} + \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3)\delta(\omega_{\mathbf{k}} + \omega_1 - \omega_2 - \omega_3)\mathrm{d}\mathbf{k}_1 \mathrm{d}\mathbf{k}_2 \mathrm{d}\mathbf{k}_3$$
$$\Gamma_{\mathbf{k}} = \pi g^2 \int |T_{0123}|^2 (N_1 N_2 + N_1 N_3 - N_2 N_3) \tag{4}$$
$$\times \delta(\mathbf{k} + \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3)\delta(\omega_{\mathbf{k}} + \omega_1 - \omega_2 - \omega_3)\mathrm{d}\mathbf{k}_1 \mathrm{d}\mathbf{k}_2 \mathrm{d}\mathbf{k}_3$$

Split S_{nl} into two terms (N.N. Ivenskikh approach based on WRT-algorithm)

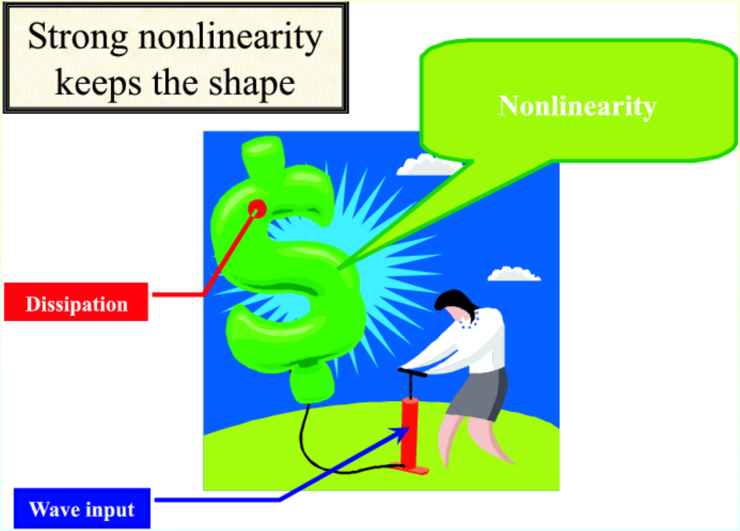


- S_{nl} consists of two great terms of opposite signatures – forcing and damping (Hasselmann mentioned this feature);
- S_{nl} is small due to proximity to an inherent state !

Scheme N 1 of spectral balance

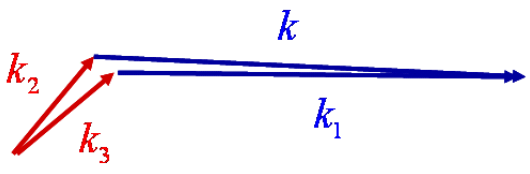


Scheme N 2 of spectral balance



An analytical estimate of Γ_{nl}

Hypothesis: The key contribution to Γ_{nl} is from interactions of pairs of long and short waves

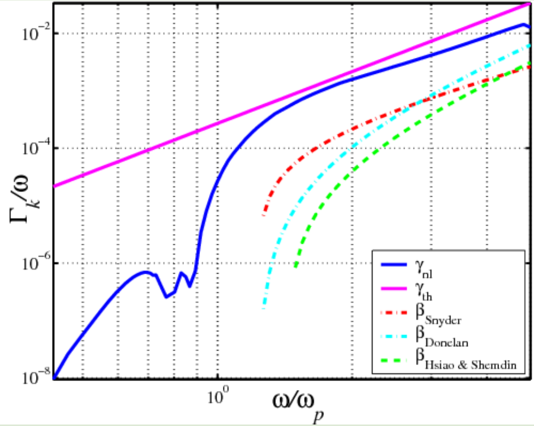


$$\Gamma_{\mathbf{k}} = 36\pi\omega(\omega/\omega_p)^3 \mu_p^4 \cos^2\Theta,$$

small parameter $\mu_p = \sqrt{\frac{E\omega_p^4}{g^2}}$ – wave steepness

An enhancing factor: $36\pi \approx 113.1$

Compare nonlinear damping decrement and wind input increment



S_{nl} surpasses S_{in} and S_{diss} in order of magnitude !

Mixed sea
I. R. Young, 2006, JGR

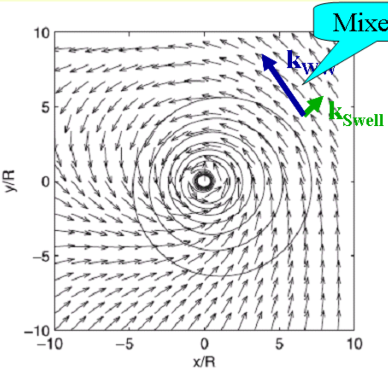


Figure 4b. A typical wind field for a Northern Hemisphere hurricane, as predicted by the model (1). The contours are of wind speed, U_{10}, with the vectors showing wind direction. The scale is identical to Figure 4a. The system is shown for the Northern Hemisphere (i.e. anti-clockwise circulation).

YOUNG: HURRICANE DIRECTIONAL SPECTRA

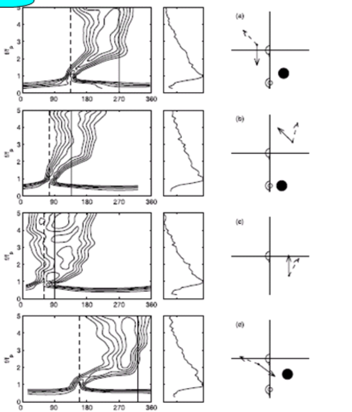


Figure 5. Examples of the directional spreading function, $D(f, \theta)$ for each quadrant of a hurricane.

Outcome:
I. R. Young, 2006, JGR

The non-linear wave-wave interaction term
is actually stronger than the representations
which are implemented in,
even the most sophisticated, research models

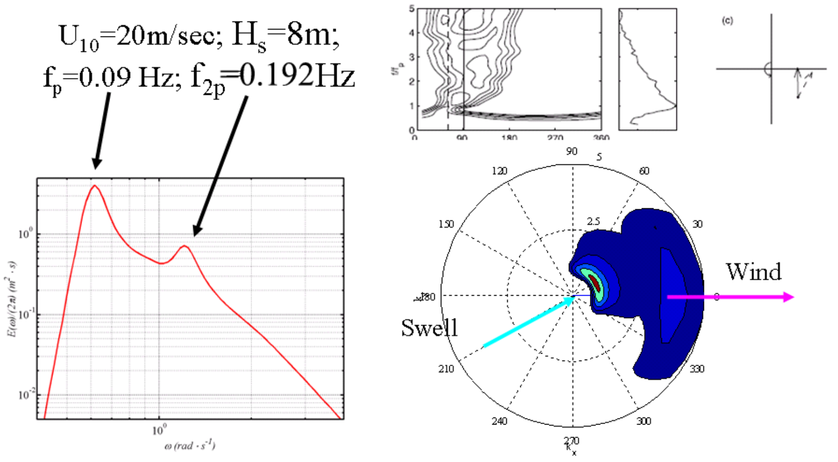
"The least sophisticated" model of the mixed sea (WISE-2008)

Nothing special: solve numerically spatially homogeneous kinetic equation (the Hasselmann equation)

$$\frac{\partial N}{\partial t} = S_{nl} + S_{in} + S_{diss}$$

with exact nonlinear transfer term S_{nl}

Case 2 by Young. Close directions of swell and wind waves. Simulations



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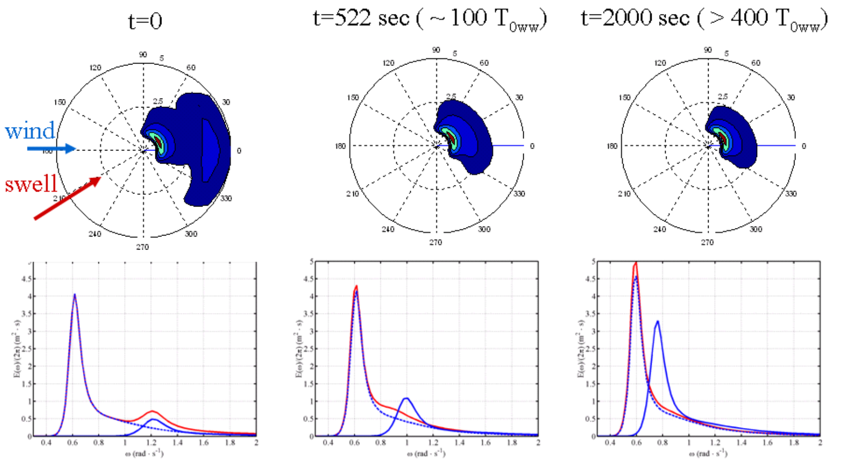


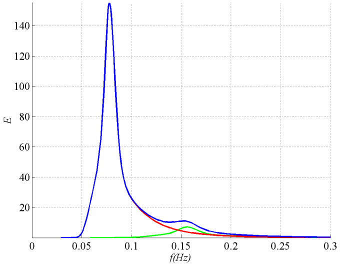
Steering, absorbing, reduction of direct wind input *etc.*

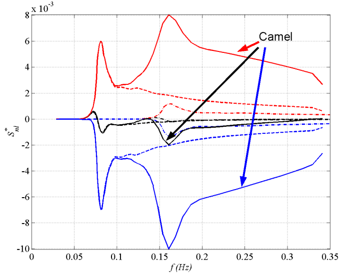
An example: Mixed sea

Can we explain the fast transition within our simple estimate of nonlinear damping?

$$\Gamma_{\mathbf{k}} = 36\pi\omega(\omega/\omega_p)^3\mu_p^4\cos^2\Theta,$$

Explanation: High (ω/ω_p) – from swell peak, high μ_p – from wind waves





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2. Interactions of long and short waves play key role in this relaxation;
3. We do not ignore wave input and dissipation, we just put them into proper place