Turbulent Quasi-Breathers in MMT Model

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Majda-McLaughlin-Tabak Model (1998):

$$i\frac{\partial\psi}{\partial t} = \left|\frac{\partial}{\partial x}\right|^{\alpha}\psi + \lambda\left|\frac{\partial}{\partial x}\right|^{\beta/4}\left(\left|\left|\frac{\partial}{\partial x}\right|^{\beta/4}\psi\right|^{2}\left|\frac{\partial}{\partial x}\right|^{\beta/4}\psi\right), \quad \lambda = \pm 1, \quad -\infty < x < \infty, \quad 0 < t < \infty$$

$$\left|\frac{\partial}{\partial x}\right|^{\alpha}\psi = \int |k|^{\alpha}\psi_k e^{-ikx}dk$$

$$\psi_k = \frac{1}{2\pi}\int \psi(x)e^{-ikx}dx$$

Nonlinear Shroedinger Equation Case:

$\alpha = 2$ $\beta = 0$

$$i\frac{\partial \psi}{\partial t} = -\frac{\partial^2 \psi}{\partial x^2} + \lambda|\psi|^2\psi$$

MMT model in integral form

$$i\frac{\partial \psi_k}{\partial t} = |k|^{\alpha}\psi_k + \lambda \int T_{kk_1k_2k_3}\psi^{\star}_{k_1}\psi_{k_2}\psi_{k_3}\delta_{k+k_1+k_2+k_3}dk_1dk_2dk_3$$

$$T_{kk_1k_2k_3} = |k|^{\beta/4}|k_1|^{\beta/4}|k_2|^{\beta/4}|k_3|^{\beta/4}$$

Zakharov equation

$$\eta(x,t) = \frac{1}{2\pi}\int e^{ikt}k^{1/4}(\psi_k + \psi^{*}_k)dk$$

$$i\frac{\partial \psi_k}{\partial t} = \frac{\delta H}{\delta \psi_k^*}$$

$$N = \int |\psi_k|^2 \, dk$$

$$H = \int |k|^\alpha |\psi_k|^2 dk + \frac{1}{2} \int T_{kk_1k_2k_3} \psi_k^\star \psi_{k_1} \psi_{k_2} \psi_{k_3} \delta_{k+k_1-k_2-k_3} dk dk_1 dk_2 dk_3$$

$$P = \frac{i}{2} \int \left(\psi \frac{\partial \psi^\star}{\partial x} - \frac{\partial \psi}{\partial x} \psi^\star \right) dx$$

Solitons and quasi-solitons

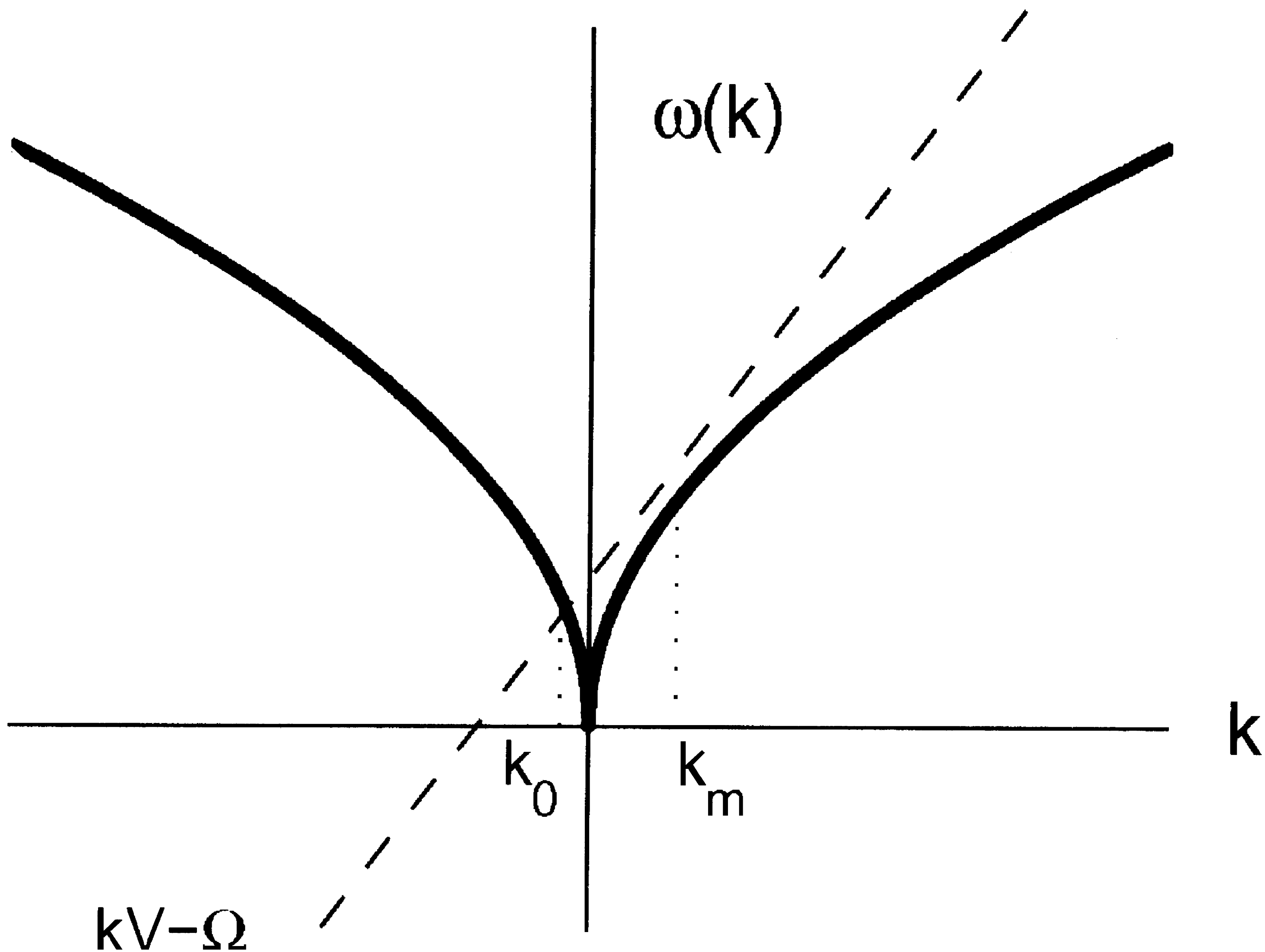
$$\psi_k = e^{i(\Omega - kV)t}\phi_k$$

$$\phi_k = -\lambda \frac{\int T_{1234}\phi_1^\star \phi_2 \phi_3 \delta(k + k_1 - k_2 - k_3) dk_1 dk_2 dk_3}{-\Omega + kV - |k|^\alpha}$$

$$\alpha = 2, \lambda = -1, T = 1, \Omega > 0 \text{ and } V^2 < 4\Omega$$

$$\phi_k = -\frac{\sqrt{\pi}}{2} \frac{1}{\cosh \frac{\pi(k - \frac{V}{2})}{2a}}, \quad a^2 = \Omega - \frac{V^2}{4}$$

Dispersion relation



Approximate MMT solution

$$\frac{q}{k_m} << 1$$

$$\psi(x,t) = \frac{1}{2k_m^{9/4}} \frac{q}{\cosh q(x - Vt)} e^{i\Omega t + ik_m(x - Vt)}$$

$$k_0 = -(3 - \sqrt{8}) \cdot k_m$$

Numerical simulation initial condition

$$\psi(x,0) = \frac{q}{2k_m^{9/4}} \frac{e^{ikx}}{\cosh qx}$$

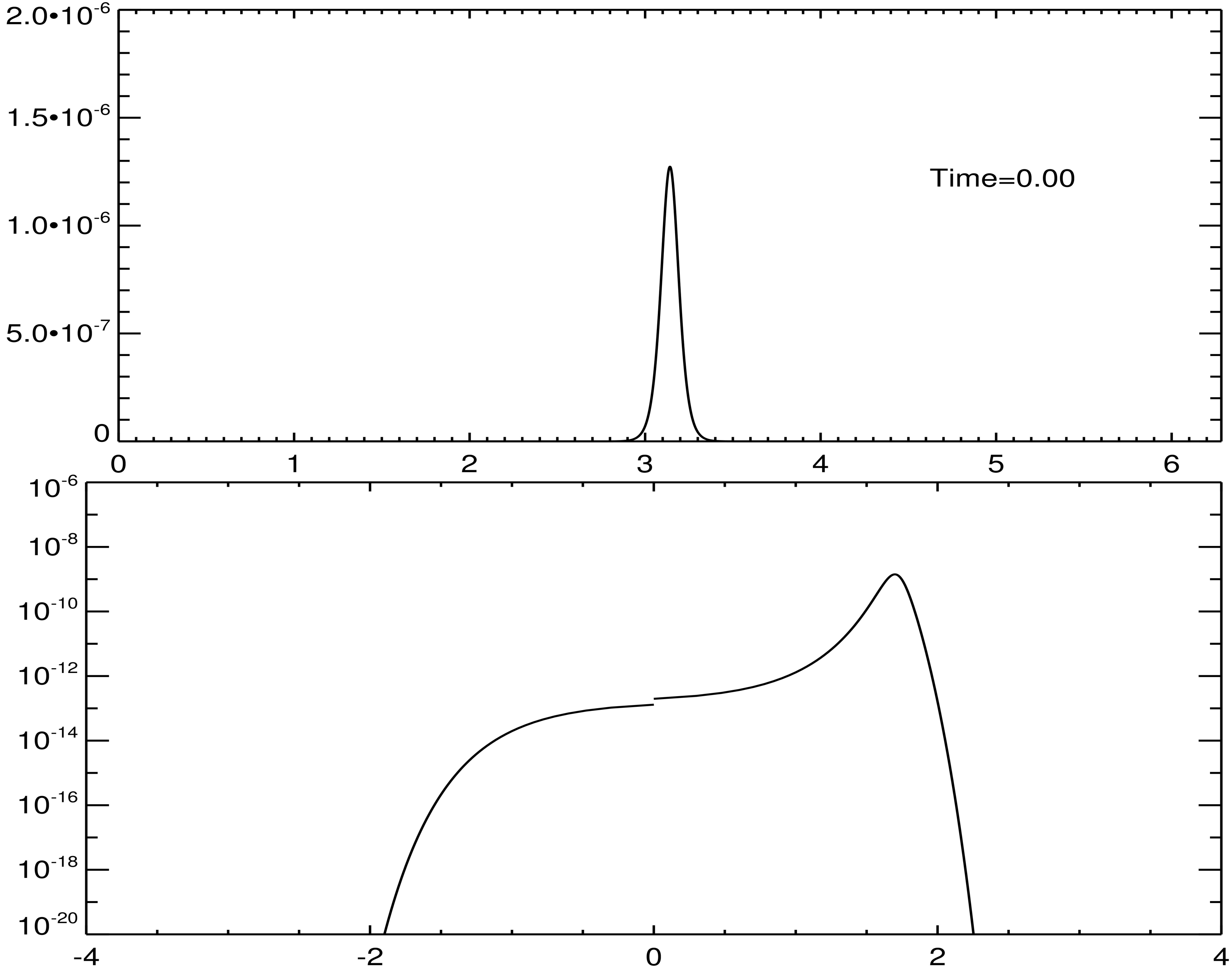
$$k_m = 50$$

$$k_0 = -(3-\sqrt{8}) \cdot k_m \simeq -8.6$$

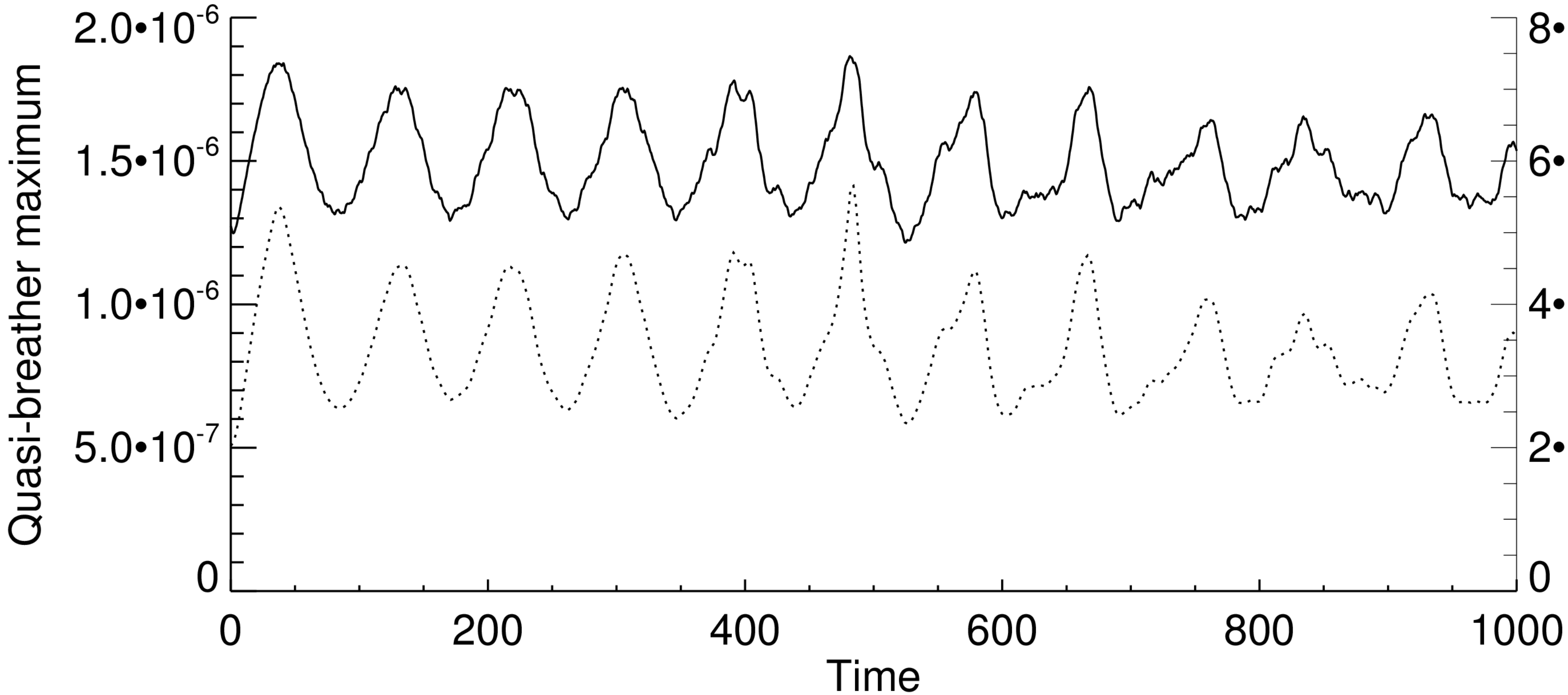
Quasi-soliton and quasi-breather regimes

$$q/k_m \lesssim 0.1$$

$$q/k_m = 0.3$$

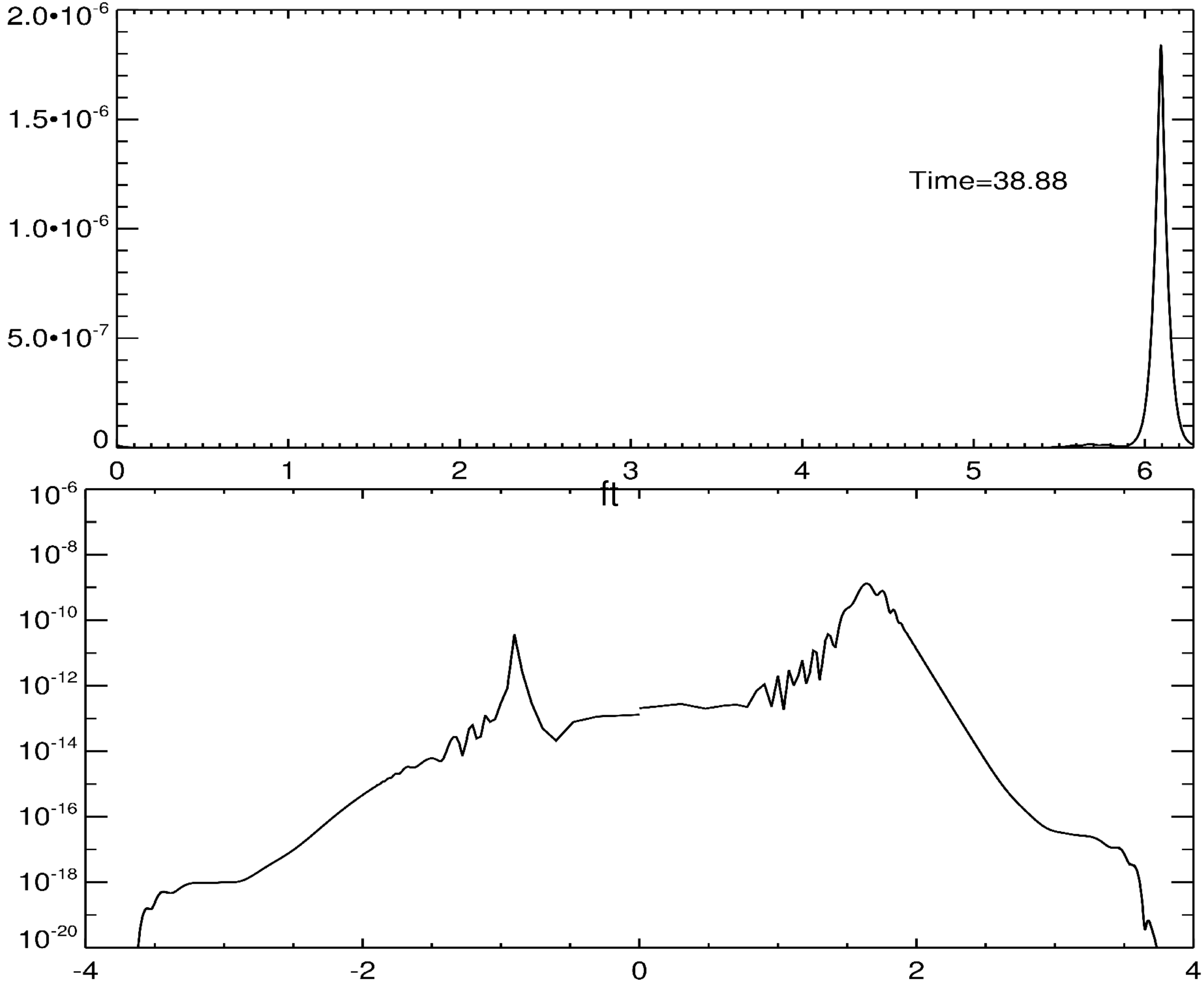


2.0•10^-6
1.5•10^-6
1.0•10^-6
5.0•10^-7
0
Time=0.00
0
1
2
3
4
5
6
10^-6
10^-8
10^-10
10^-12
10^-14
10^-16
10^-18
10^-20
-4
-2
0
2
4



$$\int \left(k-k_0\right)^2 \left(\psi\right)^2 dk$$

$$k_0 = \frac{\int k\left(\psi_k\right)^2 dk}{\int \left(\psi_{k^2}\right) dk}$$

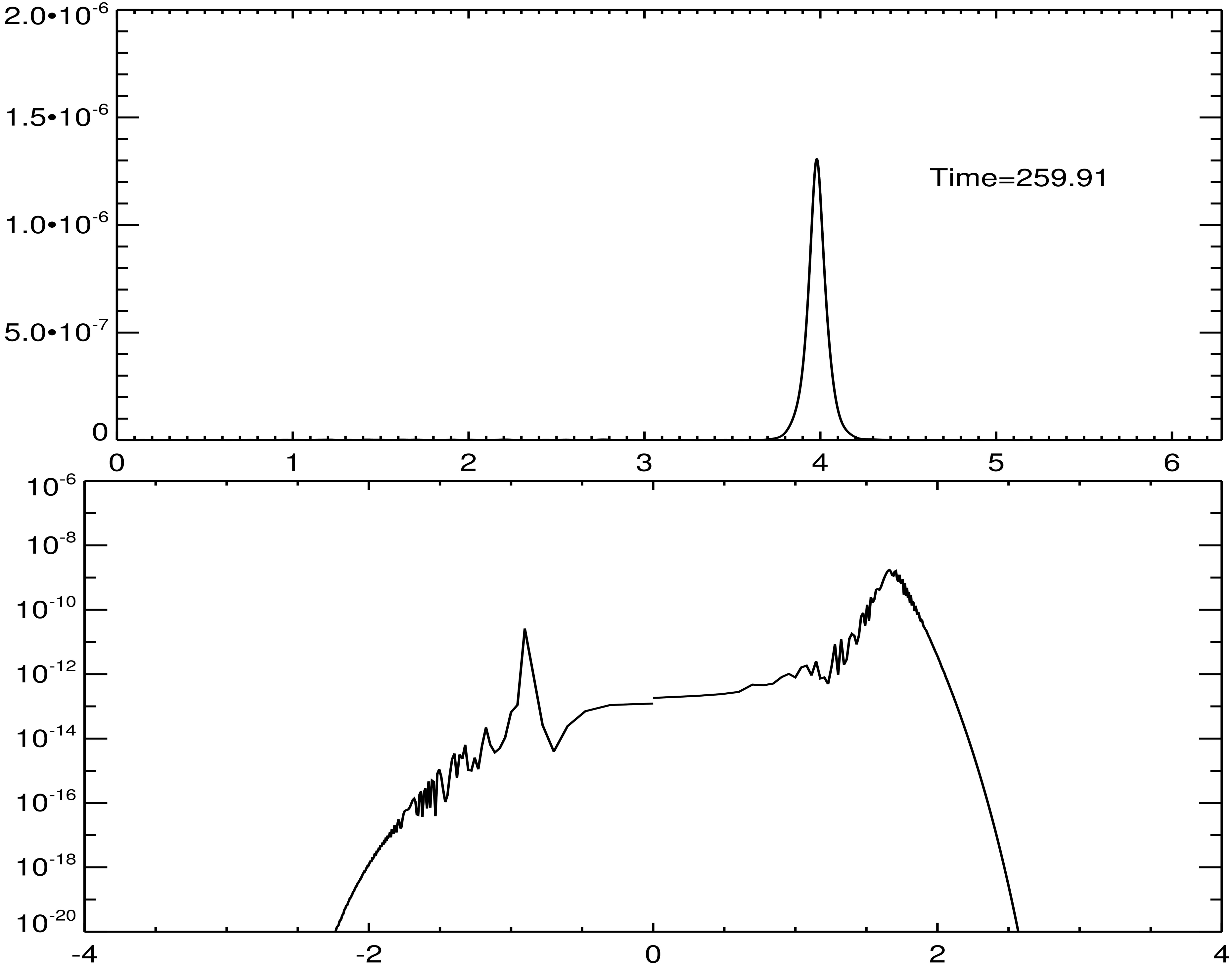


Left slope: $k^{-3.3}$ Right slope: $k^{-6.8}$

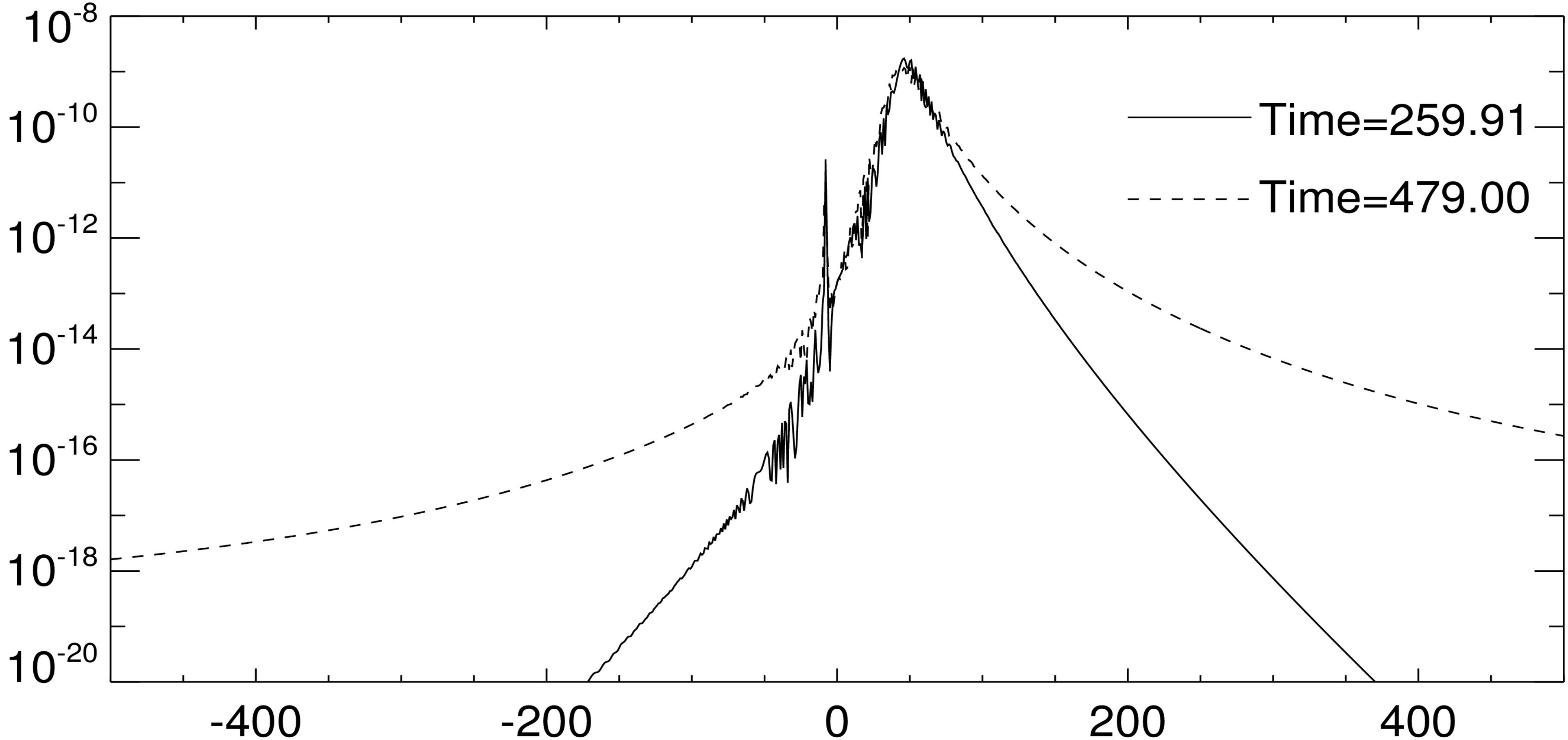
Self-Similar Solution

$$\psi(x,t)=(t_0-t)^{5/2}F\left(\frac{x}{t_0-t}\right)$$

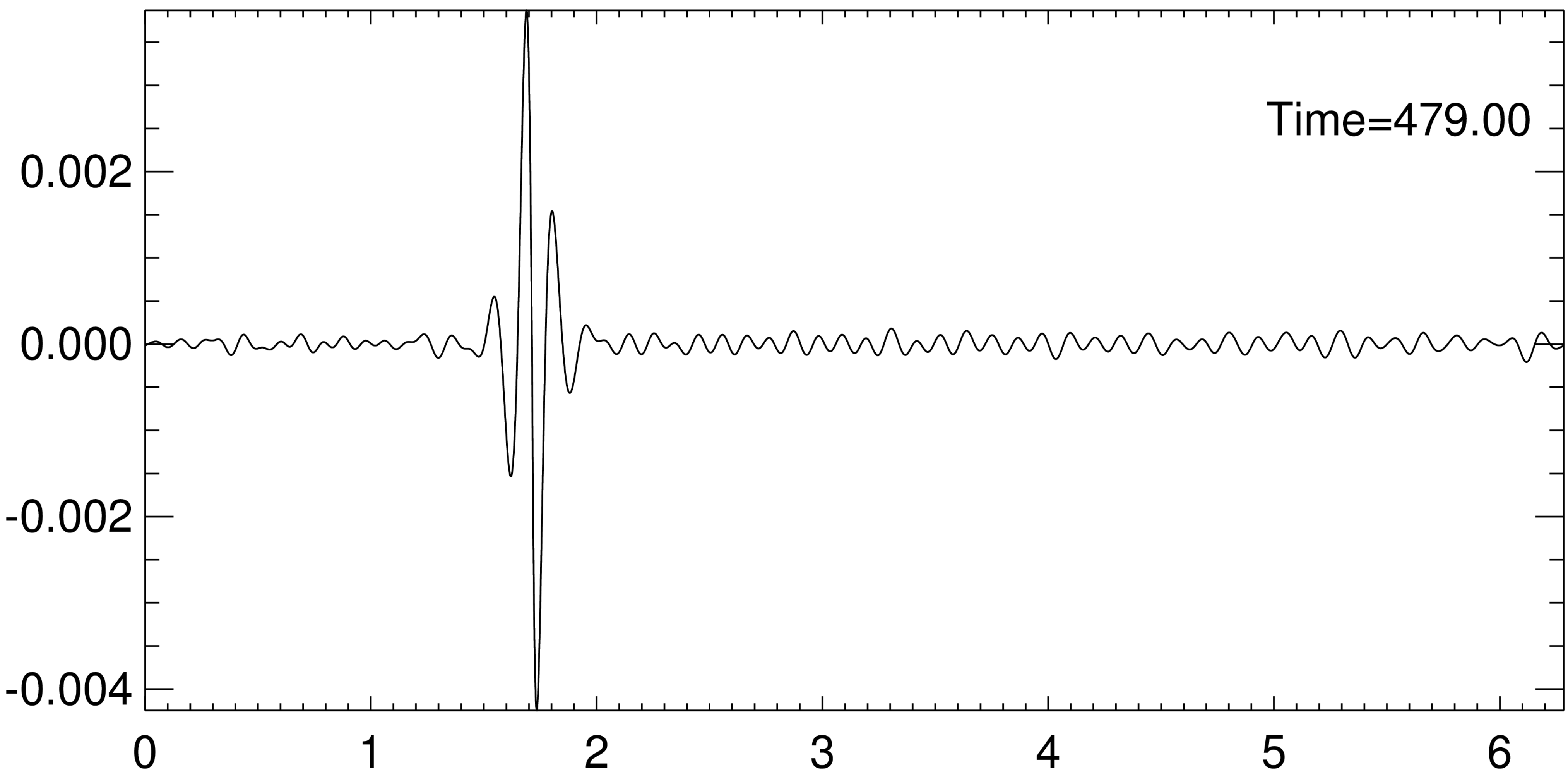
$$|\psi(k,t)|^2\simeq k^{-9/2}$$



Time=259.91



10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-18}
10^{-20}
-400
-200
0
200
400
Time=259.91
Time=479.00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0.002
0.000
-0.002
-0.004
0
1
2
3
4
5
6